PROBABILITY DISTRIBUTIONS

Anthony Phan, May 12, 2010

1. Continuous Probability distributions[1]

`lnGamma(double` x`)`. Logarithm of the Eulerian Gamma function, $\ln(\Gamma(x))$.

`Gamma(double` x`)`. Eulerian Gamma function

$$\mathbb{R}_+^* \longrightarrow [\sqrt{\pi}, +\infty[, \qquad x \longmapsto \Gamma(x) = \int_0^{+\infty} t^{x-1}\,\mathrm{e}^{-t}\,\mathrm{d}t.$$

Remember that for $n \in \mathbb{N}$, $n! = \Gamma(n+1)$.

`Beta(double` x`, double` y`)`. Eulerian Beta function

$$(\mathbb{R}_+^*)^2 \longrightarrow \mathbb{R}_+^*, \qquad x \longmapsto B(x,y) = \int_0^1 u^{x-1}(1-u)^{y-1}\,\mathrm{d}u = \frac{\Gamma(x)\times\Gamma(y)}{\Gamma(x+y)}.$$

`gammapdf(double` x`, double` a`)`. Probability density function of the Gamma distribution with parameter $a > 0$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathbb{1}_{\mathbb{R}_+}(x)\,x^{a-1}\,\mathrm{e}^{-x}/\Gamma(a).$$

`gammacdf(double` x`, double` a`)`. Cumulative distribution function of the Gamma distribution with parameter $a > 0$.

$$\mathbb{R} \longrightarrow [0,1[, \qquad x \longmapsto \mathbb{1}_{\mathbb{R}_+}(x) \int_0^x t^{a-1}\,\mathrm{e}^{-t}\,\frac{\mathrm{d}t}{\Gamma(a)}.$$

`gammaicdf(double` p`, double` a`)`. Inverse cumulative distribution function of the Gamma distribution with parameter $a > 0$.

$$[0,1[\longrightarrow \mathbb{R}_+, \qquad p \longmapsto \mathrm{gammaicdf}(p,a).$$

It is set to 0 for $p <$ `accuracy` and to `infinity` for $p > 1 -$ `accuracy`.

`normlimit`. Numerical parameter for normal computations: if X is a random variable with law $\mathcal{N}(0,1)$, the normal distribution with mean 0 and standard deviation 1, then $\mathbb{P}\{X \geqslant \texttt{normlimit}\} = \mathbb{P}\{X \leqslant -\texttt{normlimit}\} \approx 0$. Its value is (unreasonably) set to $\texttt{normlimit} = 10$ ($\texttt{normlimit} = 4$ should be sufficient).

`normalpdf(double` x`)`. Probability density function of $\mathcal{N}(0,1)$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathrm{e}^{-x^2/2}/\sqrt{2\pi}.$$

`normalcdf (double` x`)`. Cumulative distribution function of $\mathcal{N}(0,1)$.

$$\mathbb{R} \longrightarrow]0,1[, \qquad x \longmapsto \Phi(x) = \int_{-\infty}^x \mathrm{e}^{-z^2/2}\,\frac{\mathrm{d}z}{\sqrt{2\pi}} = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n\,x^{2n+1}}{(2n+1)\times n!}.$$

It is computed with its associated power series when $|x| <$ `normlimit`, and set to 0 or 1 otherwise.

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1. *See* `probability-distributions.h`

`normalcdf_(double` x`)`. Cumulative distribution function of $\mathcal{N}(0,1)$. It is just another implementation of the previous function with a Gamma cumulative distribution function since

$$\Phi(x) = \frac{1}{2}\big(\mathrm{sgn}(x) \times \mathrm{gammacdf}(x^2/2, 1/2) + 1\big), \qquad \text{for all } x \in \mathbb{R}.$$

`normalicdf(double` p`)`. Inverse cumulative distribution function of $\mathcal{N}(0,1)$.

$$]0,1[\longrightarrow \mathbb{R}, \qquad p \longmapsto \mathrm{normalicdf}(p) = \Phi^{-1}(p).$$

It is set to $\pm$`infinity` for p outside of $]$`accuracy`$, 1-$`accuracy`$[$. Of course, there is also `normalicdf_`...

Remark. — Normal distributions with mean m and standard deviation σ are not inplemented sine they an be easily derived from the standard normal distribution. For instance, one can set

```
double gaussianpdf(double x, double m, double sigma){
  return normalpdf((x-m)/sigma)/sigma;}
double gaussiancdf(double x, double m, double sigma){
  return normalcdf((x-m)/sigma);}
double gaussianicdf(double p, double m, double sigma){
  return sigma*normalicdf(p)+m;}
```

in order to get the probability, cumulative, inverse cumulative distribution functions of the $\mathcal{N}(m,\sigma^2)$ distribution with $m \in \mathbb{R}$ and $\sigma > 0$.

`chisquarepdf(double` x`, double` ν`)`. Probability density function of $\chi^2(\nu)$, the chi-square (Pearson) distribution with $\nu > 0$ degrees of freedom.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathbb{1}_{\mathbb{R}_+}(x)\,\frac{x^{\nu/2-1}\mathrm{e}^{-x/2}}{2^{\nu/2}\,\Gamma(\nu/2)}\,.$$

`chisquarecdf(double` x`, double` ν`)`. Cumulative distribution function of $\chi^2(\nu)$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathbb{1}_{\mathbb{R}_+}(x)\int_0^x t^{\nu/2-1}\mathrm{e}^{-t/2}\,\frac{\mathrm{d}t}{2^{\nu/2}\,\Gamma(\nu/2)}\,.$$

`chisquareicdf(double` p`, double` ν`)`. Inverse cumulative distribution function of $\chi^2(\nu)$.

$$[0,1[\longrightarrow \mathbb{R}_+, \qquad p \longmapsto \mathrm{chisquareicdf}(p,\nu).$$

It is set to 0 for $p <$ `accuracy` and to `infinity` for $p > 1 -$ `accuracy`.

`betapdf(double` x`, double` a`, double` b`)`. Probability density function of the Beta distribution with parameters $a >$ and $b > 0$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathbb{1}_{[0,1]}(x)\,x^{a-1}(1-x)^{b-1}/B(a,b).$$

`betacdf(double` x`, double` a`, double` b`)`. Cumulative distribution function of the Beta distribution with parameters $a > 0$ and $b > 0$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \int_0^x \mathbb{1}_{[0,1]}(u)\,u^{a-1}(1-u)^{b-1}\,\frac{\mathrm{d}u}{B(a,b)}\,.$$

`betaicdf(double` p`, double` a`, double` b`)`. Inverse cumulative distribution function of the Beta distribution with parameters $a > 0$ and $b > 0$.

$$[0,1] \longrightarrow \mathbb{R}_+, \qquad p \longmapsto \mathrm{betaicdf}(p,a,b).$$

It is set to 0 for $p <$ `accuracy` and to 1 for $p > 1 -$ `accuracy`.

`studentpdf(double` x`, double` ν`).` Probability density function of $\mathcal{T}(\nu)$, the Student distribution with $\nu > 0$ degrees of freedom.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \frac{1}{\sqrt{\nu}\, B(1/2, \nu/2)} \left(1 + \frac{x^2}{\nu}\right)^{-(\nu+1)/2}.$$

`studentcdf(double` x`, double` ν`).` Cumulative distribution function of $\mathcal{T}(\nu)$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \int_{-\infty}^{x} \left(1 + \frac{z^2}{\nu}\right)^{-(\nu+1)/2} \frac{\mathrm{d}z}{\sqrt{\nu}\, B(1/2, \nu/2)}.$$

`studenticdf(double` p`, double` ν`).` Inverse cumulative distribution function of $\mathcal{T}(\nu)$.

$$]0, 1[\longrightarrow \mathbb{R}, \qquad p \longmapsto \mathrm{studenticdf}(p, \nu).$$

It is set to $\pm$`infinity` for p outside of $]$`accuracy`$, 1 -$ `accuracy`$[$.

`fisherpdf(double` x`, double` ν_1`, double` ν_2`).` Probability density function of $\mathcal{F}(\nu_1, \nu_2)$, the Fisher distribution with $\nu_1 > 0$ and $\nu_2 > 0$ degrees of freedom (ν_1 is the numerator degree of freedom, ν_2 the demominator degree of freedom).

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathbb{1}_{\mathbb{R}_+}(x) \, \frac{(\nu_1/\nu_2)^{\nu_1/2} \, x^{\nu_1/2-1}}{B(\nu_1/2, \nu_2/2)(1 + x \times \nu_1/\nu_2)^{(\nu_1+\nu_2)/2}}.$$

`fishercdf(double` x`, double` ν_1`, double` ν_2`).` Cumulative distribution function of $\mathcal{F}(\nu_1, \nu_2)$.

$$\mathbb{R} \longrightarrow \mathbb{R}_+, \qquad x \longmapsto \mathbb{1}_{\mathbb{R}_+}(x) \int_0^x \frac{(\nu_1/\nu_2)^{\nu_1/2} \, z^{\nu_1/2-1}}{B(\nu_1/2, \nu_2/2)(1 + z \times \nu_1/\nu_2)^{(\nu_1+\nu_2)/2}} \, \mathrm{d}z.$$

`fishericdf(double` p`, double` ν_1`, double` ν_2`).` Inverse cumulative distribution function of $\mathcal{F}(\nu_1, \nu_2)$.

$$[0, 1[\longrightarrow \mathbb{R}_+, \qquad p \longmapsto \mathrm{fishericdf}(p, \nu_1, \nu_2).$$

It is set to 0 for $p <$ `accuracy` and to `infinity` for $p > 1 -$ `accuracy`.

2. Discrete Probability distributions[2]

About quantiles, please note that $q_p = k + 0.5$ when $F(k) = p$ for $k \in \mathbb{N}$.

`poissonpdf(double` x`, double` λ`).` Probability distribution function of $\mathcal{P}(\lambda)$, the Poisson distribution with parameter $\lambda \geqslant 0$.

$$\mathbb{R} \longrightarrow [0, 1], \qquad x \longmapsto \begin{cases} \mathrm{e}^{-\lambda} \, \lambda^x / x! & \text{if } x \in \mathbb{N}, \\ 0 & \text{otherwise.} \end{cases}$$

`poissoncdf(double` x`, double` λ`).` Cumulative distribution function of $\mathcal{P}(\lambda)$.

`poissonicdf(double` p`, double` λ`).` Inverse cumulative distribution function of $\mathcal{P}(\lambda)$.

`binomialpdf(double` x`, int` n`, double` π`).` Probability distribution function of $\mathcal{B}(n, \pi)$, the binomial distribution with parameters $n \in \mathbb{N}^*$ and $\pi \in [0, 1]$.

$$\mathbb{R} \longrightarrow [0, 1], \qquad x \longmapsto \begin{cases} C_n^x \, \pi^x (1 - \pi)^{n-x} & \text{if } x \in \{0, 1, \ldots, n\}, \\ 0 & \text{otherwise.} \end{cases}$$

`binomialcdf(double` x`, int` n`, double` π`).` Cumulative distribution function of $\mathcal{B}(n, \pi)$.

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`binomialicdf(double` p`, int` n`, double` π`)`. Inverse cumulative distribution function of $\mathcal{B}(n,\pi)$.

`geometricpdf(double` x`, double` π`)`. Probability distribution function of $\mathcal{G}(\pi)$, the geometrical distribution with parameter $\pi \in [0,1]$. Describe the law of the first success rank in an infinitely repeated Bernoulli trial with parameter $\pi \in [0,1]$. Thus, it is given by

$$\mathbb{R} \longrightarrow [0,1], \qquad x \longmapsto \begin{cases} \pi(1-\pi)^{x-1} & \text{if } x \in \{1,2,3,\ldots\}, \\ 0 & \text{otherwise.} \end{cases}$$

`geometriccdf(double` x`, double` π`)`. Cumulative distribution function of $\mathcal{G}(\pi)$. It returns the sum up to x of the previous probabilities. Thus it is given by

$$\mathbb{R} \longrightarrow [0,1], \qquad x \longmapsto \begin{cases} 1-(1-\pi)^{\operatorname{floor} x} & \text{if } x \geqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

`geometricicdf(double` p`, double` π`)`. Inverse cumulative distribution function of $\mathcal{G}(\pi)$.

$$[0,1] \longrightarrow \{1, 1.5, 2, 2.5, 3, 3.5, \ldots\}, \qquad p \longmapsto \operatorname{geometricicdf}(p,\pi).$$

`negativebinomialpdf(double` x`, int` n`, double` π`)`. Probability distribution function of the negative binomial ditribution with parameters $n \in \mathbb{N}^*$ and $\pi \in [0,1]$. Describe the law of the n-th success rank, $n \in \mathbb{N}^*$, in an infinitely repeated Bernoulli trial with parameter $\pi \in [0,1]$. Thus, it is given by

$$\mathbb{R} \longrightarrow [0,1], \qquad x \longmapsto \begin{cases} C_{x-1}^{n-1}\, \pi^n (1-\pi)^{x-n} & \text{if } x \in \{n, n+1, n+2, \ldots\}, \\ 0 & \text{otherwise.} \end{cases}$$

One can remark that the negative binomial distributions for $n = 1$ are just the corresponding geometric ones. It is certainly better when $n = 1$ to use functions related to geometric distributions instead of negative binomial distributions related ones.

`negativebinomialcdf(double` x`, int` n`, double` π`)`. Cumulative distribution function of the negative binomial ditribution with parameters $n \in \mathbb{N}^*$ and $\pi \in [0,1]$. One can easily prove that it is given by

$$\mathbb{R} \longrightarrow [0,1], \qquad x \longmapsto 1 - \operatorname{binomialcdf}(n-1, \operatorname{floor} x, \pi).$$

`negativebinomialicdf(double` p`, int` n`, double` π`)`. Inverse cumulative distribution function of the negative binomial ditribution with parameters $n \in \mathbb{N}^*$ and $\pi \in [0,1]$.

$$[0,1] \longrightarrow \{n, n+0.5, n+1, n+1.5, \ldots\}, \qquad p \longmapsto \operatorname{negativebinomialicdf}(p,n,\pi).$$

`hypergeometricpdf(double` x`, int` N`, int` n`, double` π`)`. Probability distribution function of $\mathcal{H}(N,n,\pi)$, the hypergeometrical distribution with parameters $N \geqslant n \in \mathbb{N}^*$ and $\pi \in [0,1]$. One should have $N\pi \in \mathbb{N}$.

$$\mathbb{R} \longrightarrow [0,1],$$
$$x \longmapsto \begin{cases} \dfrac{C_{N\pi}^{x}\, C_{N(1-\pi)}^{n-x}}{C_N^n} & \text{if } x \in \{\max(0, n-N(1-\pi)), \ldots, \min(n, N\pi)\}, \\ 0 & \text{otherwise.} \end{cases}$$

`hypergeometriccdf(double` x`, int` N`, int` n`, double` π`)`. Cumulative distribution function of $\mathcal{H}(N,n,\pi)$.

`hypergeometricicdf(double` p`, int` N`, int` n`, double` π`)`. Inverse cumulative distribution function of $\mathcal{H}(N,n,\pi)$.

3. Rather specific Probability distributions[3]

`Kolmogorovcdf(double` x`, int` n`).` Cumulative distribution function of Kolmogorov distributions. This the famous probability distributions involved in Kolmogorov–Smirnov (two-sided) Goodness of Fit tests with statistic

$$K_n = \|F - F_n\|_\infty = \sup_{x\in\mathbb{R}} \bigl|F(x) - F_n(x)\bigr| = \max_{i=1}^n \bigl(F(X_{(i)}) - (i-1)/n\bigr) \vee \bigl(i/n - F(X_{(i)})\bigr).$$

Their computation is based on "Evaluating Kolmogorov's Distribution" by George Marsaglia and Wai Wan Tsang.

`kolmogorovicdf(double` p`, int` n`).` Inverse cumulative distribution function of Kolmogorov distributions. (Please do not use it since it is based on the bisection method or dichotomy.)

`klmcdf(double` x`, int` n`).` Cumulative distribution function of the limiting distributions associated to Kolmogorov distribution by Dudley's asymptotic formula (1964):

$$\lim_{n\to\infty} \mathbb{P}\bigl\{K_n \leqslant u/\sqrt{n}\bigr\} = 1 + 2\sum_{k=1}^{\infty} (-1)^k \exp\bigl(-2k^2u^2\bigr),$$

with some numerical adjustments (Stephens M.A., 1970).

`klmicdf(double` p`, int` n`).` Inverse cumulative distribution function of the previous distributions.

`kpmcdf(double` x`, int` n`).` Cumulative distribution function of the distribution involved in the one sided Goodness of Fit tests with statistic

$$K_n^+ = \sup_{x\in\mathbb{R}} \bigl(F(x) - F_n(x)\bigr) = \max_{1\leqslant i\leqslant n} \Bigl(\frac{i}{n} - F\bigl(X_{(i)}\bigr)\Bigr)$$

or

$$K_n^- = \sup_{x\in\mathbb{R}} \bigl(F_n(x) - F(x)\bigr) = \max_{1\leqslant i\leqslant n} \Bigl(F\bigl(X_{(i)}\bigr) - \frac{i-1}{n}\Bigr)$$

which share the same distribution: for $x \in [0,1]$,

$$\begin{aligned} \texttt{kpmcdf}(x,n) = \mathbb{P}\bigl\{K_n^\pm \leqslant x\bigr\} &= x \sum_{0\leqslant k\leqslant nx} \binom{n}{k} (k/n - x)^k (x + 1 - k/n)^{n-k-1} \\ &= 1 - x \sum_{nx<k\leqslant n} \binom{n}{k} (k/n - x)^k (x + 1 - k/n)^{n-k-1}, \end{aligned}$$

the second formula being the one used for computations.

`kpmicdf(double` p`, int` n`).` Inverse cumulative distribution function of the previous distributions.

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3. *See* `probability-distributions.h`

Appendix: tables[4]

NORMALCDF

This table provides $\Phi(z)$ for $0 \leqslant z < 3$, where Φ is the cumulative distribution function of te standard normal distribution.

z	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,0	0,5000	0,5040	0,5080	0,5120	0,5160	0,5199	0,5239	0,5279	0,5319	0,5359
0,1	0,5398	0,5438	0,5478	0,5517	0,5557	0,5596	0,5636	0,5675	0,5714	0,5753
0,2	0,5793	0,5832	0,5871	0,5910	0,5948	0,5987	0,6026	0,6064	0,6103	0,6141
0,3	0,6179	0,6217	0,6255	0,6293	0,6331	0,6368	0,6406	0,6443	0,6480	0,6517
0,4	0,6554	0,6591	0,6628	0,6664	0,6700	0,6736	0,6772	0,6808	0,6844	0,6879
0,5	0,6915	0,6950	0,6985	0,7019	0,7054	0,7088	0,7123	0,7157	0,7190	0,7224
0,6	0,7257	0,7291	0,7324	0,7357	0,7389	0,7422	0,7454	0,7486	0,7517	0,7549
0,7	0,7580	0,7611	0,7642	0,7673	0,7704	0,7734	0,7764	0,7794	0,7823	0,7852
0,8	0,7881	0,7910	0,7939	0,7967	0,7995	0,8023	0,8051	0,8078	0,8106	0,8133
0,9	0,8159	0,8186	0,8212	0,8238	0,8264	0,8289	0,8315	0,8340	0,8365	0,8389
1,0	0,8413	0,8438	0,8461	0,8485	0,8508	0,8531	0,8554	0,8577	0,8599	0,8621
1,1	0,8643	0,8665	0,8686	0,8708	0,8729	0,8749	0,8770	0,8790	0,8810	0,8830
1,2	0,8849	0,8869	0,8888	0,8907	0,8925	0,8944	0,8962	0,8980	0,8997	0,9015
1,3	0,9032	0,9049	0,9066	0,9082	0,9099	0,9115	0,9131	0,9147	0,9162	0,9177
1,4	0,9192	0,9207	0,9222	0,9236	0,9251	0,9265	0,9279	0,9292	0,9306	0,9319
1,5	0,9332	0,9345	0,9357	0,9370	0,9382	0,9394	0,9406	0,9418	0,9429	0,9441
1,6	0,9452	0,9463	0,9474	0,9484	0,9495	0,9505	0,9515	0,9525	0,9535	0,9545
1,7	0,9554	0,9564	0,9573	0,9582	0,9591	0,9599	0,9608	0,9616	0,9625	0,9633
1,8	0,9641	0,9649	0,9656	0,9664	0,9671	0,9678	0,9686	0,9693	0,9699	0,9706
1,9	0,9713	0,9719	0,9726	0,9732	0,9738	0,9744	0,9750	0,9756	0,9761	0,9767
2,0	0,9772	0,9778	0,9783	0,9788	0,9793	0,9798	0,9803	0,9808	0,9812	0,9817
2,1	0,9821	0,9826	0,9830	0,9834	0,9838	0,9842	0,9846	0,9850	0,9854	0,9857
2,2	0,9861	0,9864	0,9868	0,9871	0,9875	0,9878	0,9881	0,9884	0,9887	0,9890
2,3	0,9893	0,9896	0,9898	0,9901	0,9904	0,9906	0,9909	0,9911	0,9913	0,9916
2,4	0,9918	0,9920	0,9922	0,9925	0,9927	0,9929	0,9931	0,9932	0,9934	0,9936
2,5	0,9938	0,9940	0,9941	0,9943	0,9945	0,9946	0,9948	0,9949	0,9951	0,9952
2,6	0,9953	0,9955	0,9956	0,9957	0,9959	0,9960	0,9961	0,9962	0,9963	0,9964
2,7	0,9965	0,9966	0,9967	0,9968	0,9969	0,9970	0,9971	0,9972	0,9973	0,9974
2,8	0,9974	0,9975	0,9976	0,9977	0,9977	0,9978	0,9979	0,9979	0,9980	0,9981
2,9	0,9981	0,9982	0,9982	0,9983	0,9984	0,9984	0,9985	0,9985	0,9986	0,9986

4. *See* `probability-tables.c`

NORMALICDF

This table provides $\Phi^{-1}(\alpha)$ for $0.5 \leqslant \alpha < 1$. Note that $\Phi^{-1}(1-\alpha) = -\Phi^{-1}(\alpha)$.

α	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,50	0,0000	0,0251	0,0502	0,0753	0,1004	0,1257	0,1510	0,1764	0,2019	0,2275
0,60	0,2533	0,2793	0,3055	0,3319	0,3585	0,3853	0,4125	0,4399	0,4677	0,4959
0,70	0,5244	0,5534	0,5828	0,6128	0,6433	0,6745	0,7063	0,7388	0,7722	0,8064
0,80	0,8416	0,8779	0,9154	0,9542	0,9945	1,0364	1,0803	1,1264	1,1750	1,2265
0,90	1,2816	1,3408	1,4051	1,4758	1,5548	1,6449	1,7507	1,8808	2,0537	2,3263

α	0,990	0,991	0,992	0,993	0,994	0,995	0,996	0,997	0,998	0,999
$\Phi^{-1}(\alpha)$	2,3263	2,3656	2,4089	2,4573	2,5121	2,5758	2,6521	2,7478	2,8782	3,0902

α	0,9990	0,9991	0,9992	0,9993	0,9994	0,9995	0,9996	0,9997	0,9998	0,9999
$\Phi^{-1}(\alpha)$	3,0902	3,1214	3,1559	3,1947	3,2389	3,2905	3,3528	3,4316	3,5401	3,7190

NORMALICDFBIS

This table provides $\Phi^{-1}(1-\alpha/2)$ for $0 < \alpha < 1$.

α	0,00	0,01	0,02	0,03	0,04	0,05	0,06	0,07	0,08	0,09
0,00	∞	2,5758	2,3263	2,1701	2,0537	1,9600	1,8808	1,8119	1,7507	1,6954
0,10	1,6449	1,5982	1,5548	1,5141	1,4758	1,4395	1,4051	1,3722	1,3408	1,3106
0,20	1,2816	1,2536	1,2265	1,2004	1,1750	1,1503	1,1264	1,1031	1,0803	1,0581
0,30	1,0364	1,0152	0,9945	0,9741	0,9542	0,9346	0,9154	0,8965	0,8779	0,8596
0,40	0,8416	0,8239	0,8064	0,7892	0,7722	0,7554	0,7388	0,7225	0,7063	0,6903
0,50	0,6745	0,6588	0,6433	0,6280	0,6128	0,5978	0,5828	0,5681	0,5534	0,5388
0,60	0,5244	0,5101	0,4959	0,4817	0,4677	0,4538	0,4399	0,4261	0,4125	0,3989
0,70	0,3853	0,3719	0,3585	0,3451	0,3319	0,3186	0,3055	0,2924	0,2793	0,2663
0,80	0,2533	0,2404	0,2275	0,2147	0,2019	0,1891	0,1764	0,1637	0,1510	0,1383
0,90	0,1257	0,1130	0,1004	0,0878	0,0753	0,0627	0,0502	0,0376	0,0251	0,0125

NORMALICDFTER

This table provides $\Phi^{-1}(1-\alpha/2)$ for very small values of α.

α	0,001	0,0001	1e-05	1e-06	1e-07	1e-08	1e-09
$z_{1-\alpha/2}$	3,2905	3,8906	4,4172	4,8916	5,3267	5,7307	6,1094

CHISQUAREICDF

This table provides chisquareicdf$(1-\alpha, \nu)$.

ν \ α	0,990	0,975	0,950	0,900	0,100	0,050	0,025	0,010	0,001
1	0,0002	0,0010	0,0039	0,0158	2,7055	3,8415	5,0239	6,6349	10,8276
2	0,0201	0,0506	0,1026	0,2107	4,6052	5,9915	7,3778	9,2103	13,8155
3	0,1148	0,2158	0,3518	0,5844	6,2514	7,8147	9,3484	11,3449	16,2662
4	0,2971	0,4844	0,7107	1,0636	7,7794	9,4877	11,1433	13,2767	18,4668
5	0,5543	0,8312	1,1455	1,6103	9,2364	11,0705	12,8325	15,0863	20,5150
6	0,8721	1,2373	1,6354	2,2041	10,6446	12,5916	14,4494	16,8119	22,4577
7	1,2390	1,6899	2,1673	2,8331	12,0170	14,0671	16,0128	18,4753	24,3219
8	1,6465	2,1797	2,7326	3,4895	13,3616	15,5073	17,5345	20,0902	26,1245
9	2,0879	2,7004	3,3251	4,1682	14,6837	16,9190	19,0228	21,6660	27,8772
10	2,5582	3,2470	3,9403	4,8652	15,9872	18,3070	20,4832	23,2093	29,5883
11	3,0535	3,8157	4,5748	5,5778	17,2750	19,6751	21,9200	24,7250	31,2641
12	3,5706	4,4038	5,2260	6,3038	18,5493	21,0261	23,3367	26,2170	32,9095
13	4,1069	5,0088	5,8919	7,0415	19,8119	22,3620	24,7356	27,6882	34,5282
14	4,6604	5,6287	6,5706	7,7895	21,0641	23,6848	26,1189	29,1412	36,1233
15	5,2293	6,2621	7,2609	8,5468	22,3071	24,9958	27,4884	30,5779	37,6973
16	5,8122	6,9077	7,9616	9,3122	23,5418	26,2962	28,8454	31,9999	39,2524
17	6,4078	7,5642	8,6718	10,0852	24,7690	27,5871	30,1910	33,4087	40,7902
18	7,0149	8,2307	9,3905	10,8649	25,9894	28,8693	31,5264	34,8053	42,3124
19	7,6327	8,9065	10,1170	11,6509	27,2036	30,1435	32,8523	36,1909	43,8202
20	8,2604	9,5908	10,8508	12,4426	28,4120	31,4104	34,1696	37,5662	45,3147
21	8,8972	10,2829	11,5913	13,2396	29,6151	32,6706	35,4789	38,9322	46,7970
22	9,5425	10,9823	12,3380	14,0415	30,8133	33,9244	36,7807	40,2894	48,2679
23	10,1957	11,6886	13,0905	14,8480	32,0069	35,1725	38,0756	41,6384	49,7282
24	10,8564	12,4012	13,8484	15,6587	33,1962	36,4150	39,3641	42,9798	51,1786
25	11,5240	13,1197	14,6114	16,4734	34,3816	37,6525	40,6465	44,3141	52,6197
26	12,1981	13,8439	15,3792	17,2919	35,5632	38,8851	41,9232	45,6417	54,0520
27	12,8785	14,5734	16,1514	18,1139	36,7412	40,1133	43,1945	46,9629	55,4760
28	13,5647	15,3079	16,9279	18,9392	37,9159	41,3371	44,4608	48,2782	56,8923
29	14,2565	16,0471	17,7084	19,7677	39,0875	42,5570	45,7223	49,5879	58,3012
30	14,9535	16,7908	18,4927	20,5992	40,2560	43,7730	46,9792	50,8922	59,7031

STUDENTICDF

This table provides studenticdf$(1-\alpha/2,\nu)$.

ν \ α	0,900	0,500	0,300	0,200	0,100	0,050	0,020	0,010	0,001
1	0,1584	1,0000	1,9626	3,0777	6,3138	12,7062	31,8205	63,6567	636,6192
2	0,1421	0,8165	1,3862	1,8856	2,9200	4,3027	6,9646	9,9248	31,5991
3	0,1366	0,7649	1,2498	1,6377	2,3534	3,1824	4,5407	5,8409	12,9240
4	0,1338	0,7407	1,1896	1,5332	2,1318	2,7764	3,7469	4,6041	8,6103
5	0,1322	0,7267	1,1558	1,4759	2,0150	2,5706	3,3649	4,0321	6,8688
6	0,1311	0,7176	1,1342	1,4398	1,9432	2,4469	3,1427	3,7074	5,9588
7	0,1303	0,7111	1,1192	1,4149	1,8946	2,3646	2,9980	3,4995	5,4079
8	0,1297	0,7064	1,1081	1,3968	1,8595	2,3060	2,8965	3,3554	5,0413
9	0,1293	0,7027	1,0997	1,3830	1,8331	2,2622	2,8214	3,2498	4,7809
10	0,1289	0,6998	1,0931	1,3722	1,8125	2,2281	2,7638	3,1693	4,5869
11	0,1286	0,6974	1,0877	1,3634	1,7959	2,2010	2,7181	3,1058	4,4370
12	0,1283	0,6955	1,0832	1,3562	1,7823	2,1788	2,6810	3,0545	4,3178
13	0,1281	0,6938	1,0795	1,3502	1,7709	2,1604	2,6503	3,0123	4,2208
14	0,1280	0,6924	1,0763	1,3450	1,7613	2,1448	2,6245	2,9768	4,1405
15	0,1278	0,6912	1,0735	1,3406	1,7531	2,1314	2,6025	2,9467	4,0728
16	0,1277	0,6901	1,0711	1,3368	1,7459	2,1199	2,5835	2,9208	4,0150
17	0,1276	0,6892	1,0690	1,3334	1,7396	2,1098	2,5669	2,8982	3,9651
18	0,1274	0,6884	1,0672	1,3304	1,7341	2,1009	2,5524	2,8784	3,9216
19	0,1274	0,6876	1,0655	1,3277	1,7291	2,0930	2,5395	2,8609	3,8834
20	0,1273	0,6870	1,0640	1,3253	1,7247	2,0860	2,5280	2,8453	3,8495
21	0,1272	0,6864	1,0627	1,3232	1,7207	2,0796	2,5176	2,8314	3,8193
22	0,1271	0,6858	1,0614	1,3212	1,7171	2,0739	2,5083	2,8188	3,7921
23	0,1271	0,6853	1,0603	1,3195	1,7139	2,0687	2,4999	2,8073	3,7676
24	0,1270	0,6848	1,0593	1,3178	1,7109	2,0639	2,4922	2,7969	3,7454
25	0,1269	0,6844	1,0584	1,3163	1,7081	2,0595	2,4851	2,7874	3,7251
26	0,1269	0,6840	1,0575	1,3150	1,7056	2,0555	2,4786	2,7787	3,7066
27	0,1268	0,6837	1,0567	1,3137	1,7033	2,0518	2,4727	2,7707	3,6896
28	0,1268	0,6834	1,0560	1,3125	1,7011	2,0484	2,4671	2,7633	3,6739
29	0,1268	0,6830	1,0553	1,3114	1,6991	2,0452	2,4620	2,7564	3,6594
30	0,1267	0,6828	1,0547	1,3104	1,6973	2,0423	2,4573	2,7500	3,6460
40	0,1265	0,6807	1,0500	1,3031	1,6839	2,0211	2,4233	2,7045	3,5510
80	0,1261	0,6776	1,0432	1,2922	1,6641	1,9901	2,3739	2,6387	3,4163
120	0,1259	0,6765	1,0409	1,2886	1,6577	1,9799	2,3578	2,6174	3,3735
∞	0,1257	0,6745	1,0364	1,2816	1,6449	1,9600	2,3263	2,5758	3,2905

FISHERICDF

This table provides `fishericdf`$(1-\alpha, \nu_1, \nu_2)$ with $\alpha = 0.05$. Values are roughly rounded.

$\nu_2 \backslash \nu_1$	1	2	3	4	5	6	8	10	15	20	30
1	161	200	216	225	230	234	239	242	246	248	250
2	18,5	19,0	19,2	19,2	19,3	19,3	19,4	19,4	19,4	19,4	19,5
3	10,1	9,55	9,28	9,12	9,01	8,94	8,85	8,79	8,70	8,66	8,62
4	7,71	6,94	6,59	6,39	6,26	6,16	6,04	5,96	5,86	5,80	5,75
5	6,61	5,79	5,41	5,19	5,05	4,95	4,82	4,74	4,62	4,56	4,50
6	5,99	5,14	4,76	4,53	4,39	4,28	4,15	4,06	3,94	3,87	3,81
7	5,59	4,74	4,35	4,12	3,97	3,87	3,73	3,64	3,51	3,44	3,38
8	5,32	4,46	4,07	3,84	3,69	3,58	3,44	3,35	3,22	3,15	3,08
9	5,12	4,26	3,86	3,63	3,48	3,37	3,23	3,14	3,01	2,94	2,86
10	4,96	4,10	3,71	3,48	3,33	3,22	3,07	2,98	2,85	2,77	2,70
11	4,84	3,98	3,59	3,36	3,20	3,09	2,95	2,85	2,72	2,65	2,57
12	4,75	3,89	3,49	3,26	3,11	3,00	2,85	2,75	2,62	2,54	2,47
13	4,67	3,81	3,41	3,18	3,03	2,92	2,77	2,67	2,53	2,46	2,38
14	4,60	3,74	3,34	3,11	2,96	2,85	2,70	2,60	2,46	2,39	2,31
15	4,54	3,68	3,29	3,06	2,90	2,79	2,64	2,54	2,40	2,33	2,25
16	4,49	3,63	3,24	3,01	2,85	2,74	2,59	2,49	2,35	2,28	2,19
17	4,45	3,59	3,20	2,96	2,81	2,70	2,55	2,45	2,31	2,23	2,15
18	4,41	3,55	3,16	2,93	2,77	2,66	2,51	2,41	2,27	2,19	2,11
19	4,38	3,52	3,13	2,90	2,74	2,63	2,48	2,38	2,23	2,16	2,07
20	4,35	3,49	3,10	2,87	2,71	2,60	2,45	2,35	2,20	2,12	2,04
22	4,30	3,44	3,05	2,82	2,66	2,55	2,40	2,30	2,15	2,07	1,98
24	4,26	3,40	3,01	2,78	2,62	2,51	2,36	2,25	2,11	2,03	1,94
26	4,23	3,37	2,98	2,74	2,59	2,47	2,32	2,22	2,07	1,99	1,90
28	4,20	3,34	2,95	2,71	2,56	2,45	2,29	2,19	2,04	1,96	1,87
30	4,17	3,32	2,92	2,69	2,53	2,42	2,27	2,16	2,01	1,93	1,84
40	4,08	3,23	2,84	2,61	2,45	2,34	2,18	2,08	1,92	1,84	1,74
50	4,03	3,18	2,79	2,56	2,40	2,29	2,13	2,03	1,87	1,78	1,69
60	4,00	3,15	2,76	2,53	2,37	2,25	2,10	1,99	1,84	1,75	1,65
80	3,96	3,11	2,72	2,49	2,33	2,21	2,06	1,95	1,79	1,70	1,60
100	3,94	3,09	2,70	2,46	2,31	2,19	2,03	1,93	1,77	1,68	1,57

KOLMOGOROVICDF

This table provides `kolmogorovicdf`$(1-\alpha, n)$.

α \ n	0	1	2	3	4	5	6	7	8	9
0,01	1,0000	0,9950	0,9293	0,8290	0,7342	0,6685	0,6166	0,5758	0,5418	0,5133
0,05	1,0000	0,9750	0,8419	0,7076	0,6239	0,5633	0,5193	0,4834	0,4543	0,4300
0,10	1,0000	0,9500	0,7764	0,6360	0,5652	0,5094	0,4680	0,4361	0,4096	0,3875
0,15	1,0000	0,9250	0,7261	0,5958	0,5248	0,4744	0,4353	0,4050	0,3806	0,3601
0,20	1,0000	0,9000	0,6838	0,5648	0,4927	0,4470	0,4104	0,3815	0,3583	0,3391

α \ n	10	11	12	13	14	15	16	17	18	19
0,01	0,4889	0,4677	0,4490	0,4325	0,4176	0,4042	0,3920	0,3809	0,3706	0,3612
0,05	0,4092	0,3912	0,3754	0,3614	0,3489	0,3376	0,3273	0,3180	0,3094	0,3014
0,10	0,3687	0,3524	0,3381	0,3255	0,3142	0,3040	0,2947	0,2863	0,2785	0,2714
0,15	0,3425	0,3273	0,3141	0,3023	0,2918	0,2823	0,2737	0,2659	0,2587	0,2520
0,20	0,3226	0,3083	0,2957	0,2847	0,2748	0,2658	0,2577	0,2503	0,2436	0,2373

α \ n	20	21	22	23	24	25	26	27	28	29
0,01	0,3524	0,3443	0,3367	0,3295	0,3229	0,3166	0,3106	0,3050	0,2997	0,2947
0,05	0,2941	0,2872	0,2809	0,2749	0,2693	0,2640	0,2591	0,2544	0,2499	0,2457
0,10	0,2647	0,2586	0,2528	0,2475	0,2424	0,2377	0,2332	0,2290	0,2250	0,2212
0,15	0,2459	0,2402	0,2348	0,2298	0,2251	0,2207	0,2166	0,2127	0,2089	0,2054
0,20	0,2315	0,2261	0,2211	0,2164	0,2120	0,2079	0,2040	0,2003	0,1968	0,1934

α \ n	30	31	32	33	34	35	36	37	38	39
0,01	0,2899	0,2853	0,2809	0,2768	0,2728	0,2690	0,2653	0,2618	0,2584	0,2552
0,05	0,2417	0,2379	0,2342	0,2308	0,2274	0,2242	0,2212	0,2183	0,2154	0,2127
0,10	0,2176	0,2141	0,2108	0,2077	0,2047	0,2018	0,1991	0,1965	0,1939	0,1915
0,15	0,2021	0,1989	0,1958	0,1929	0,1901	0,1875	0,1849	0,1825	0,1801	0,1779
0,20	0,1903	0,1873	0,1844	0,1817	0,1791	0,1766	0,1742	0,1718	0,1696	0,1675

α \ n	40	42	44	46	48	50	52	54	56	58
0,01	0,2521	0,2461	0,2406	0,2354	0,2306	0,2260	0,2217	0,2177	0,2138	0,2102
0,05	0,2101	0,2052	0,2006	0,1963	0,1922	0,1884	0,1848	0,1814	0,1782	0,1752
0,10	0,1891	0,1847	0,1805	0,1766	0,1730	0,1696	0,1664	0,1633	0,1604	0,1577
0,15	0,1757	0,1715	0,1677	0,1641	0,1607	0,1575	0,1545	0,1517	0,1490	0,1465
0,20	0,1654	0,1616	0,1579	0,1545	0,1514	0,1484	0,1456	0,1429	0,1404	0,1380

α \ n	60	65	70	75	80	85	90	95	100	105
0,01	0,2067	0,1988	0,1917	0,1853	0,1795	0,1742	0,1694	0,1649	0,1608	0,1570
0,05	0,1723	0,1657	0,1597	0,1544	0,1496	0,1452	0,1412	0,1375	0,1340	0,1308
0,10	0,1551	0,1491	0,1438	0,1390	0,1347	0,1307	0,1271	0,1238	0,1207	0,1178
0,15	0,1441	0,1385	0,1336	0,1291	0,1251	0,1214	0,1181	0,1150	0,1121	0,1094
0,20	0,1357	0,1305	0,1258	0,1216	0,1178	0,1144	0,1112	0,1083	0,1056	0,1031

α \ n	110	120	130	140	150	160	170	180	190	200
0,01	0,1534	0,1470	0,1413	0,1362	0,1316	0,1275	0,1237	0,1203	0,1171	0,1142
0,05	0,1279	0,1225	0,1178	0,1135	0,1097	0,1063	0,1031	0,1003	0,0976	0,0952
0,10	0,1151	0,1103	0,1060	0,1022	0,0988	0,0957	0,0929	0,0903	0,0879	0,0857
0,15	0,1070	0,1025	0,0985	0,0950	0,0918	0,0889	0,0863	0,0839	0,0817	0,0796
0,20	0,1008	0,0965	0,0928	0,0895	0,0865	0,0838	0,0813	0,0790	0,0769	0,0750

KLMICDF

This table provides `klmicdf`$(1-\alpha, n)$, the usual asymptotic approximation of the Kolmogorov inverse cunmulative distribution function.

α \ n	0	1	2	3	4	5	6	7	8	9
0,01	1,0000	1,0000	1,0000	0,8497	0,7483	0,6767	0,6226	0,5798	0,5448	0,5156
0,05	1,0000	1,0000	0,8425	0,7090	0,6244	0,5646	0,5195	0,4838	0,4546	0,4302
0,10	1,0000	0,9950	0,7592	0,6389	0,5627	0,5088	0,4681	0,4359	0,4097	0,3877
0,15	1,0000	0,9252	0,7059	0,5941	0,5232	0,4731	0,4353	0,4053	0,3809	0,3605
0,20	1,0000	0,8722	0,6655	0,5600	0,4932	0,4460	0,4103	0,3821	0,3591	0,3398

α \ n	10	11	12	13	14	15	16	17	18	19
0,01	0,4907	0,4691	0,4501	0,4333	0,4183	0,4047	0,3924	0,3812	0,3709	0,3614
0,05	0,4094	0,3914	0,3756	0,3616	0,3490	0,3377	0,3274	0,3181	0,3095	0,3015
0,10	0,3690	0,3527	0,3385	0,3258	0,3145	0,3043	0,2951	0,2866	0,2789	0,2717
0,15	0,3431	0,3280	0,3147	0,3030	0,2925	0,2830	0,2744	0,2665	0,2593	0,2526
0,20	0,3234	0,3092	0,2967	0,2856	0,2757	0,2668	0,2586	0,2512	0,2444	0,2382

α \ n	20	21	22	23	24	25	26	27	28	29
0,01	0,3525	0,3444	0,3367	0,3296	0,3228	0,3165	0,3106	0,3050	0,2996	0,2946
0,05	0,2942	0,2873	0,2810	0,2750	0,2694	0,2641	0,2591	0,2545	0,2500	0,2458
0,10	0,2651	0,2589	0,2532	0,2478	0,2428	0,2380	0,2335	0,2293	0,2253	0,2215
0,15	0,2465	0,2408	0,2354	0,2304	0,2257	0,2213	0,2171	0,2132	0,2095	0,2059
0,20	0,2324	0,2270	0,2219	0,2172	0,2128	0,2086	0,2047	0,2010	0,1975	0,1941

α \ n	30	31	32	33	34	35	36	37	38	39
0,01	0,2898	0,2852	0,2808	0,2766	0,2726	0,2688	0,2652	0,2616	0,2583	0,2550
0,05	0,2418	0,2379	0,2343	0,2308	0,2275	0,2243	0,2212	0,2183	0,2155	0,2128
0,10	0,2179	0,2144	0,2111	0,2080	0,2050	0,2021	0,1994	0,1967	0,1942	0,1917
0,15	0,2026	0,1994	0,1963	0,1934	0,1906	0,1879	0,1854	0,1829	0,1806	0,1783
0,20	0,1910	0,1880	0,1851	0,1823	0,1797	0,1772	0,1748	0,1724	0,1702	0,1681

α \ n	40	42	44	46	48	50	52	54	56	58
0,01	0,2519	0,2459	0,2404	0,2353	0,2304	0,2259	0,2216	0,2175	0,2137	0,2100
0,05	0,2102	0,2052	0,2006	0,1963	0,1923	0,1885	0,1849	0,1815	0,1783	0,1752
0,10	0,1894	0,1849	0,1808	0,1769	0,1732	0,1698	0,1666	0,1635	0,1607	0,1579
0,15	0,1761	0,1720	0,1681	0,1645	0,1611	0,1579	0,1549	0,1521	0,1494	0,1468
0,20	0,1660	0,1621	0,1585	0,1551	0,1519	0,1489	0,1460	0,1433	0,1408	0,1384

α \ n	60	65	70	75	80	85	90	95	100	105
0,01	0,2065	0,1986	0,1915	0,1851	0,1793	0,1740	0,1692	0,1648	0,1607	0,1568
0,05	0,1723	0,1657	0,1598	0,1545	0,1496	0,1452	0,1412	0,1375	0,1341	0,1309
0,10	0,1553	0,1493	0,1440	0,1392	0,1348	0,1309	0,1272	0,1239	0,1208	0,1179
0,15	0,1444	0,1388	0,1339	0,1294	0,1254	0,1217	0,1183	0,1152	0,1123	0,1097
0,20	0,1361	0,1309	0,1262	0,1220	0,1182	0,1147	0,1115	0,1086	0,1059	0,1034

α \ n	110	120	130	140	150	160	170	180	190	200
0,01	0,1533	0,1468	0,1411	0,1361	0,1315	0,1274	0,1236	0,1202	0,1170	0,1141
0,05	0,1279	0,1225	0,1178	0,1135	0,1097	0,1063	0,1031	0,1003	0,0976	0,0952
0,10	0,1153	0,1104	0,1061	0,1023	0,0989	0,0958	0,0929	0,0904	0,0880	0,0858
0,15	0,1072	0,1027	0,0987	0,0951	0,0919	0,0891	0,0864	0,0840	0,0818	0,0797
0,20	0,1010	0,0968	0,0930	0,0897	0,0867	0,0840	0,0815	0,0792	0,0771	0,0752